Cut problems in graph theory are extensively studied and well understood. However, cuts that partition graphs into components that are connected in a prescribed way, have received comparatively less attention. In this thesis, we formally define several connected cut problems, focusing on MINIMUM k-CONNECTED CUT and MINIMUM k-CONNECTED CUT WITH SOURCE problems. Given a graph $G = (V, E)$, and an interger k, the goal is to partition a vertex set V into two subsets, S and $V \setminus S$, such that the induced subgraph $G[S]$ is connected, the size of S is exactly k, and the number of edges between S and $V \setminus S$ is minimized. In the second problem, we are in addition given a source vertex $s \in V$ that has to be included in S. Our first result is a proof that both MINIMUM k-CONNECTED CUT and MINIMUM k-CONNECTED CUT WITH SOURCE problems are NP-complete. Additionally, we propose a bicriteria approximation algorithm for the MINIMUM k-CONNECTED CUT WITH SOURCE problem. As a result, we also obtain a bicriteria approximation for the general MINIMUM k-CONNECTED CUT problem.