

$$\min \sum_{e \in E} \sum_{i=1}^k x_e^i \quad (1a)$$

$$x_{uv}^i \geq f_u^i - f_v^j \quad \forall \{uv\} \in E \ \forall i \neq j \in [k] \quad (1b)$$

$$x_{uv}^i \geq f_v^j - f_u^i \quad \forall \{uv\} \in E \ \forall i \neq j \in [k] \quad (1c)$$

$$\sum_{v \in V, \{s_i, v\} \in E} f_{s_i v}^i = n_i - 1 \quad \forall i \in [k] \quad (1d)$$

$$f_{s_i}^i = 1 \quad \forall i \in [k] \quad (1e)$$

$$\sum_{u \in V, \{u, v\} \in E} f_{uv}^i = \sum_{u \in V, \{v, u\} \in E} f_{vu}^i + f_v^i \quad \forall v \in V \setminus \{s\} \ \forall i \in [k] \quad (1f)$$

$$\sum_{u \in V} f_u^i = n_i \quad \forall i \in [k] \quad (1g)$$

$$(n - k) \cdot f_v^i \geq \sum_{u \in V, \{uv\} \in E} f_{uv}^i \quad \forall v \in V \setminus \{s\} \ \forall i \in [k] \quad (1h)$$

$$\sum_{i=1}^k n_i = n \quad (1i)$$

$$\sum_{i=1}^k f_v^i = 1 \quad \forall v \in V \quad (1j)$$

$$f_v^i \in \{0, 1\} \quad \forall v \in V \ \forall i \in [k] \quad (1k)$$

$$f_{uv}^i \in \mathbb{N}^+ \quad \forall \{u, v\} \in E \ \forall i \in [k] \quad (1l)$$

$$x_{uv}^i \in \{0, 1\} \quad \forall \{u, v\} \in E \quad (1m)$$

$$\min c \quad (2a)$$

$$\sum_{e \in E} x_e^i \leq c \quad \forall i \in [k] \quad (2b)$$

$$x_{uv}^i \geq f_u^i - f_v^j \quad \forall \{uv\} \in E \ \forall i \neq j \in [k] \quad (2c)$$

$$x_{uv}^i \geq f_v^j - f_u^i \quad \forall \{uv\} \in E \ \forall i \neq j \in [k] \quad (2d)$$

$$\sum_{v \in V, \{s_i, v\} \in E} f_{s_i v}^i = n_i - 1 \quad \forall i \in [k] \quad (2e)$$

$$f_{s_i}^i = 1 \quad \forall i \in [k] \quad (2f)$$

$$\sum_{u \in V, \{u, v\} \in E} f_{uv}^i = \sum_{u \in V, \{v, u\} \in E} f_{vu}^i + f_v^i \quad \forall v \in V \setminus \{s\} \ \forall i \in [k] \quad (2g)$$

$$\sum_{u \in V} f_u^i = n_i \quad \forall i \in [k] \quad (2h)$$

$$(n - k) \cdot f_v^i \geq \sum_{u \in V, \{uv\} \in E} f_{uv}^i \quad \forall v \in V \setminus \{s\} \ \forall i \in [k] \quad (2i)$$

$$\sum_{i=1}^k n_i = n \quad (2j)$$

$$\sum_{i=1}^k f_v^i = 1 \quad \forall v \in V \quad (2k)$$

$$f_v^i \in \{0, 1\} \quad \forall v \in V \ \forall i \in [k] \quad (2l)$$

$$f_{uv}^i \in \mathbb{N}^+ \quad \forall \{u, v\} \in E \ \forall i \in [k] \quad (2m)$$

$$x_{uv}^i \in \{0, 1\} \quad \forall \{u, v\} \in E \quad (2n)$$